
$(1, 2)^*$ - δ_{gp} Continuous Function in Bitopological Spaces

B. Meera Devi ^{*}
P. Subbulakshmi ^{**}
D.K. Nathan ^{***}

Abstract

The aim of this paper is to introduce a new class of functions called $(1, 2)^*$ - δ_{gp} continuous functions. We obtain several characterization and some their properties. Also we investigate its relationship with other types of functions in bitopological spaces. Further we introduce and study a new class of functions namely $(1, 2)^*$ - δ_{gp} irresolute functions.

Keywords:

$(1, 2)^*$ - δ_{gp} closed sets,
 $(1, 2)^*$ - δ_{gp} open sets,
 $(1, 2)^*$ - δ_{gp} Continuous
and $(1, 2)^*$ - δ_{gp} irresolute..

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Author correspondence:

* Assistant Professor,
Department of Mathematics,
Sri S.R.N.M.College
Sattur-626 203, Tamil Nadu, India.

1. Introduction

In 1963, Kelley [3] initiated the study of bitopological spaces. A nonempty set X equipped with two topological spaces τ_1 and τ_2 is called a bitopological spaces and is denoted by (X, τ_1, τ_2) . M. Lellis Thivagar and O.Ravi [6] introduced a new type of generalized sets called $(1, 2)^*$ -semi generalized closed sets and a new class of generalized functions called $(1, 2)^*$ -semi generalized continuous maps in 2006. S.S. Benchalli and J.B.Toranagatti [1] introduced delta generalized pre-closed sets in topological space. The purpose of this present paper is to define a new class of generalized continuous function called $(1, 2)^*$ - δ_{gp} continuous and investigate their relationships to other generalized continuous functions. We further study a new class of functions namely $(1, 2)^*$ - δ_{gp} irresolute.

2. Preliminaries

Throughout this paper (X, τ_1, τ_2) (or briefly X) represent bitopological spaces on which no separation axioms are assumed unless otherwise mentioned.

Definition 2.1. [8] A subset B of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$ -open if $B = U_1 \cup U_2$ where $U_1 \in \tau_1$ and $U_2 \in \tau_2$. The complement of $\tau_1\tau_2$ -open is called $\tau_1\tau_2$ -closed.

Remark 2.2. [8] $\tau_1\tau_2$ -open subset of X need not necessarily form a topology.

Definition 2.3. [8] A subset A of a bitopological space (X, τ_1, τ_2) is called

(i) The $\tau_1\tau_2$ -closure of A , denoted by $\tau_1\tau_2\text{-cl}(A)$ is defined by $\tau_1\tau_2$ -closure $(A) = \bigcap \{ F / A \subseteq F \text{ and } F \text{ is } \tau_1\tau_2\text{-closed} \}$.

(ii) The $\tau_1\tau_2$ -interior of A , denoted by $\tau_1\tau_2\text{-int}(A)$ is defined by $\tau_1\tau_2$ -interior $(A) = \bigcup \{ F / A \subseteq F \text{ and } F \text{ is } \tau_1\tau_2\text{-open} \}$.

Definition 2.4. A subset A of a bitopological space (X, τ_1, τ_2) is called

(i) $(1, 2)^*$ -pre-open [8] if $A \subseteq \tau_1\tau_2\text{-int}(\tau_1\tau_2\text{-cl}(A))$ and $(1, 2)^*$ -pre-closed if $\tau_1\tau_2\text{-cl}(\tau_1\tau_2\text{-int}(A)) \subseteq A$.

(ii) $(1, 2)^*$ -b open [4] if $A \subseteq (\tau_1\tau_2\text{-cl}(\tau_1\tau_2\text{-int}(A))) \cup (\tau_1\tau_2\text{-int}(\tau_1\tau_2\text{-cl}(A)))$ and $(1, 2)^*$ -b closed if $(\tau_1\tau_2\text{-cl}(\tau_1\tau_2\text{-int}(A))) \cap (\tau_1\tau_2\text{-int}(\tau_1\tau_2\text{-cl}(A))) \subseteq A$.

(iii) $(1, 2)^*$ -regular-open [12] if $A = \tau_1\tau_2\text{-int}(\tau_1\tau_2\text{-cl}(A))$ and $(1, 2)^*$ -regular closed if $A = \tau_1\tau_2\text{-cl}(\tau_1\tau_2\text{-int}(A))$.

The $(1, 2)^*$ -pre-closure of a subset A of X , denoted by $(1, 2)^*\text{-pcl}(A)$ is the intersection of all $(1, 2)^*$ -pre-closed sets containing A . The $(1, 2)^*$ -pre-interior of a subset A of X , denoted by $(1, 2)^*\text{-pint}(A)$ is the union of $(1, 2)^*$ -pre-open sets contained in A .

Definition 2.5. [9] The $(1, 2)^*$ - δ interior of a subset A of X is the union of all $(1, 2)^*$ -regular open set of X contained in A and is denoted by $(1, 2)^*\text{-}\delta\text{int}(A)$. The subset A is called $(1, 2)^*$ - δ open if $A = (1, 2)^*\text{-}\delta\text{int}(A)$, ie. a set is $(1, 2)^*$ - δ open if it is the union of $(1, 2)^*$ -regular open sets. The complement of a $(1, 2)^*$ - δ open is called $(1, 2)^*$ - δ closed. Alternatively, a set $A \subseteq (X, \tau_1, \tau_2)$ is called $(1, 2)^*$ - δ closed if $A = (1, 2)^*\text{-}\delta\text{cl}(A)$, where $(1, 2)^*\text{-}\delta\text{cl}(A) = \{ x \in X : \tau_1\tau_2\text{-int}(\tau_1\tau_2\text{-cl}(A)) \cap A = \emptyset, U \in \tau_1\tau_2 \text{ and } x \in U \}$.

Definition 2.6. A subset A of a bitopological space (X, τ_1, τ_2) is called

(i) $(1, 2)^*$ -generalized closed set (briefly $(1, 2)^*\text{-g closed}$) [11] if $\tau_1\tau_2\text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_1\tau_2$ -open in X .

(ii) $(1, 2)^*$ -generalized b-closed set (briefly $(1, 2)^*\text{-gb closed}$) [13] if $\tau_1\tau_2\text{-bcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_1\tau_2$ -open in X .

(iii) $(1, 2)^*$ -generalized pre-closed set (briefly $(1, 2)^*\text{-gp closed}$) [14] if $\tau_1\tau_2\text{-pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_1\tau_2$ -open in X .

(iv) $(1, 2)^*$ -generalized pre regular closed set (briefly $(1, 2)^*\text{-gpr closed}$) [10] if $\tau_1\tau_2\text{-pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $(1, 2)^*$ -regular open in X .

(v) $(1, 2)^*$ - δ generalized closed set (briefly $(1, 2)^*$ - δ g closed) [9] if $\tau_1\tau_2 - \delta cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $(1, 2)^*$ - open in X .

(vi) $(1, 2)^*$ - delta generalized pre-closed (briefly, $(1, 2)^*$ - δ gp closed) [9] if $\tau_1\tau_2 - pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is $(1, 2)^*$ - δ open in X .

The complement of the above mentioned closed sets are their respective open sets.

Definition 2.7. Recall that function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called

(i) $(1, 2)^*$ - continuous [7] if $f^{-1}(V)$ is $(1, 2)^*$ - closed in (X, τ_1, τ_2) for every closed V of (Y, σ_1, σ_2) .

(ii) $(1, 2)^*$ - generalized continuous (briefly $(1, 2)^*$ - g continuous) [6] if $f^{-1}(V)$ is $(1, 2)^*$ - g closed in (X, τ_1, τ_2) for every closed V of (Y, σ_1, σ_2) .

(iii) $(1, 2)^*$ - generalized b continuous (briefly $(1, 2)^*$ -gb continuous) [13] if $f^{-1}(V)$ is $(1, 2)^*$ - g closed in (X, τ_1, τ_2) for every closed V of (Y, σ_1, σ_2) .

(iii) $(1, 2)^*$ - b continuous (briefly $(1, 2)^*$ - b continuous) [2] if $f^{-1}(V)$ is $(1, 2)^*$ - b closed in (X, τ_1, τ_2) for every closed V of (Y, σ_1, σ_2) .

(iv) $(1, 2)^*$ - generalized pre continuous (briefly $(1, 2)^*$ - gp continuous) [5] if $f^{-1}(V)$ is $(1, 2)^*$ - gp closed in (X, τ_1, τ_2) for every closed V of (Y, σ_1, σ_2) .

(v) $(1, 2)^*$ - generalized pre regular continuous (briefly $(1, 2)^*$ - gpr continuous) [11] if $f^{-1}(V)$ is $(1, 2)^*$ - gpr closed in (X, τ_1, τ_2) for every closed V of (Y, σ_1, σ_2) .

3. $(1, 2)^*$ - δ gp Continuous function

We introduce the following definitions.

Definition 3.1. A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be $(1, 2)^*$ - δ gp continuous, if $f^{-1}(V)$ is $(1, 2)^*$ - δ gp closed set in X for every $\sigma_1\sigma_2$ - closed set V in Y .

Example 3.2. Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$, $\tau_2 = \{\phi, \{b\}, \{a, b\}, X\}$, $\sigma_1 = \{\phi, \{b\}, Y\}$ and $\sigma_2 = \{\phi, \{a\}, Y\}$. Let a map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be defined by $f(a) = b, f(b) = a, f(c) = c$. Then f is $(1, 2)^*$ - δ gp continuous.

Theorem 3.3. Every $(1, 2)^*$ - continuous map is $(1, 2)^*$ - δ gp continuous.

Proof. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be $(1, 2)^*$ - continuous. Let V be $\sigma_1\sigma_2$ - closed set in Y . Since f is $(1, 2)^*$ - continuous, $f^{-1}(V)$ is $\tau_1\tau_2$ - closed. But every $\tau_1\tau_2$ - closed set is $(1, 2)^*$ - δ gp closed. Therefore $f^{-1}(V)$ is $(1, 2)^*$ - δ gp closed. Hence f is $(1, 2)^*$ - δ gp continuous.

Remark 3.4. The converse of the above theorem is not true in general as shown in the following example.

Example 3.5. Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$, $\tau_2 = \{\phi, \{b\}, \{a, c\}, X\}$, $\sigma_1 = \{\phi, \{a\}, Y\}$ and $\sigma_2 = \{\phi, \{a\}, \{a, c\}, Y\}$. Let a map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be defined by $f(a) = c, f(b) = a, f(c) = b$. Clearly f is $(1, 2)^*$ - δ gp continuous but not $(1, 2)^*$ - continuous because $f^{-1}(\{b\}) = \{c\}$ is not $\tau_1\tau_2$ closed.

Theorem 3.6. Every $(1, 2)^*$ -pre continuous map is $(1, 2)^*$ - δ gp continuous.

Proof. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be $(1, 2)^*$ -pre continuous. Let V be $\sigma_1\sigma_2$ -closed set in Y . Since f is $(1, 2)^*$ -pre continuous, $f^{-1}(V)$ is $(1, 2)^*$ -pre closed. But every $(1, 2)^*$ -pre closed set is $(1, 2)^*$ - δ gp closed. Therefore $f^{-1}(V)$ is $(1, 2)^*$ - δ gp closed. Hence f is $(1, 2)^*$ - δ gp continuous.

Remark 3.7. The converse of the above theorem is not true in general as shown in the following example.

Example 3.8. Let $X = Y = \{a, b, c, d\}$, $\tau_1 = \{\phi, \{a\}, \{a, b, c\}, X\}$, $\tau_2 = \{\phi, \{b\}, \{a, b\}, X\}$, $\sigma_1 = \{\phi, \{a, d\}, \{a, c, d\}, Y\}$ and $\sigma_2 = \{\phi, \{b\}, \{b, c, d\}, Y\}$. Let a map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be defined by $f(a) = d, f(b) = c, f(c) = a$ and $f(d) = b$. Clearly f is $(1, 2)^*$ - δ gp continuous but not $(1, 2)^*$ -pre continuous because $f^{-1}(\{b, c\}) = \{b, d\}$ is not $(1, 2)^*$ -pre closed.

Theorem 3.9. Every $(1, 2)^*$ -gp continuous map is $(1, 2)^*$ - δ gp continuous.

Proof. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be $(1, 2)^*$ -gp continuous. Let V be $\sigma_1\sigma_2$ -closed set in Y . Since f is $(1, 2)^*$ -gp continuous, $f^{-1}(V)$ is $(1, 2)^*$ -gp closed. But every $(1, 2)^*$ -gp closed set is $(1, 2)^*$ - δ gp closed. Therefore $f^{-1}(V)$ is $(1, 2)^*$ - δ gp closed. Hence f is $(1, 2)^*$ - δ gp continuous.

Remark 3.10. The converse of the above theorem is not true in general as shown in the following example.

Example 3.11. Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\phi, \{b\}, \{a, b\}, X\}$, $\tau_2 = \{\phi, \{a\}, \{a, c\}, X\}$, $\sigma_1 = \{\phi, \{b, c\}, Y\}$ and $\sigma_2 = \{\phi, \{a\}, \{a, c\}, Y\}$. Let a map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be defined by $f(a) = b, f(b) = c, f(c) = a$. Clearly f is $(1, 2)^*$ - δ gp continuous but not $(1, 2)^*$ -gp continuous because $f^{-1}(\{b\}) = \{a\}$ is not $(1, 2)^*$ -gp closed.

Theorem 3.12. Every $(1, 2)^*$ - δ gp continuous is $(1, 2)^*$ -gpr continuous.

Proof. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be $(1, 2)^*$ - δ gp continuous. Let V be $\sigma_1\sigma_2$ -closed set in Y . Since f is $(1, 2)^*$ - δ gp continuous, $f^{-1}(V)$ is $(1, 2)^*$ - δ gp closed. But every $(1, 2)^*$ - δ gp closed set is $(1, 2)^*$ -gpr closed. Therefore $f^{-1}(V)$ is $(1, 2)^*$ -gpr closed. Hence f is $(1, 2)^*$ -gpr continuous.

Remark 3.13. The converse of the above theorem is not true in general as shown in the following example.

Example 3.14. Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\phi, \{b\}, \{a, c\}, X\}$, $\tau_2 = \{\phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, X\}$, $\sigma_1 = \{\phi, \{a\}, Y\}$ and $\sigma_2 = \{\phi, \{a, b\}, Y\}$. Let a map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be defined by $f(a) = b, f(b) = c, f(c) = a$. Clearly f is $(1, 2)^*$ -gpr continuous but not $(1, 2)^*$ - δ gp continuous because $f^{-1}(\{b, c\}) = \{a, b\}$ is not $(1, 2)^*$ - δ gp closed.

Remark 3.15. The following example shows that $(1, 2)^*$ - δ gp continuous function is independent of $(1, 2)^*$ -b continuous function and $(1, 2)^*$ -gb continuous function.

Example 3.16. Let $X = Y = \{a, b, c, d\}$, $\tau_1 = \{\phi, \{b\}, \{a, b\}, X\}$, $\tau_2 = \{\phi, \{a\}, \{a, b, c\}, X\}$, $\sigma_1 = \{\phi, \{b, c\}, \{a, b, c\}, Y\}$ and $\sigma_2 = \{\phi, \{c\}, \{a, c, d\}, Y\}$. Let a map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be defined by $f(a) = c, f(b) = a, f(c) = d, f(d) = b$. Clearly f is $(1, 2)^* - \delta_{gp}$ continuous but not $(1, 2)^* - gb$ continuous and also it is not $(1, 2)^* - b$ continuous because $f^{-1}(\{a, b, d\}) = \{a, b, c\}$ is not $(1, 2)^* - gb$ closed and also it is not $(1, 2)^* - b$ closed.

Example 3.17. Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$, $\tau_2 = \{\phi, \{c\}, \{a, c\}, X\}$, $\sigma_1 = \{\phi, \{b\}, \{b, c\}, Y\}$ and $\sigma_2 = \{\phi, \{a, b\}, Y\}$. Let a map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be defined by $f(a) = c, f(b) = a, f(c) = b$. Clearly f is $(1, 2)^* - gb$ and $(1, 2)^* - b$ continuous but not $(1, 2)^* - \delta_{gp}$ continuous because $f^{-1}(\{c\}) = \{a\}$ is not $(1, 2)^* - \delta_{gp}$ closed.

Remark 3.18. If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $(1, 2)^* - \delta_{gp}$ continuous and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ is $(1, 2)^* - \delta_{gp}$ continuous, then $g \circ f : (X, \tau_1, \tau_2) \rightarrow (Z, \eta_1, \eta_2)$ need not be $(1, 2)^* - \delta_{gp}$ continuous. The following example supports our claim.

Example 3.19. Let $X = Y = Z = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$, $\tau_2 = \{\phi, \{c\}, \{a, c\}, X\}$, $\sigma_1 = \{\phi, \{a\}, Y\}$, $\sigma_2 = \{\phi, \{c\}, Y\}$, $\eta_1 = \{\phi, \{a\}, Z\}$ and $\eta_2 = \{\phi, \{b\}, \{a, c\}, Z\}$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ be the identity maps. Then f and g are $(1, 2)^* - \delta_{gp}$ continuous functions, but $g \circ f$ is not $(1, 2)^* - \delta_{gp}$ continuous because $(g \circ f)^{-1}(\{a, c\}) = f^{-1}(g^{-1}(\{a, c\})) = f^{-1}(\{a, c\}) = \{a, c\}$ is not $(1, 2)^* - \delta_{gp}$ closed.

Proposition 3.20. A map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $(1, 2)^* - \delta_{gp}$ continuous if and only if $f^{-1}(U)$ is $(1, 2)^* - \delta_{gp}$ open in X for every $\sigma_1 \sigma_2$ - open set U in Y .

Proof. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be $(1, 2)^* - \delta_{gp}$ continuous and $\sigma_1 \sigma_2$ - open set U in Y . Then U^c is $\sigma_1 \sigma_2$ - closed in Y and since f is $(1, 2)^* - \delta_{gp}$ continuous, $f^{-1}(U^c)$ is $(1, 2)^* - \delta_{gp}$ closed in X . But $f^{-1}(U^c) = [f^{-1}(U)]^c$ and so $f^{-1}(U)$ is $(1, 2)^* - \delta_{gp}$ open in X .

Conversely, assume that $f^{-1}(U)$ is $(1, 2)^* - \delta_{gp}$ open in X for every $\sigma_1 \sigma_2$ - open set U in Y . Let F be a $\sigma_1 \sigma_2$ - closed set in Y . Then F^c is $\sigma_1 \sigma_2$ - open in Y and by assumption $f^{-1}(F^c)$ is $(1, 2)^* - \delta_{gp}$ open in X . Since $f^{-1}(F^c) = [f^{-1}(F)]^c$, $f^{-1}(F)$ is $(1, 2)^* - \delta_{gp}$ closed in X and so f is $(1, 2)^* - \delta_{gp}$ continuous.

Definition 3.21. A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called $(1, 2)^* - \delta_{gp}$ irresolute if $f^{-1}(V)$ is $(1, 2)^* - \delta_{gp}$ closed in (X, τ_1, τ_2) for each $(1, 2)^* - \delta_{gp}$ closed set V in (Y, σ_1, σ_2) .

Example 3.22. Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$, $\tau_2 = \{\phi, \{b\}, \{a, c\}, X\}$, $\sigma_1 = \{\phi, \{a\}, \{a, c\}, Y\}$ and $\sigma_2 = \{\phi, \{c\}, Y\}$. Let a map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be defined by $f(a) = c, f(b) = a, f(c) = b$. Hence f is $(1, 2)^* - \delta_{gp}$ irresolute.

Theorem 3.23. If a map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ called $(1, 2)^* - \delta_{gp}$ irresolute then it is $(1, 2)^* - \delta_{gp}$ continuous.

Proof. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be $(1, 2)^* - \delta_{gp}$ irresolute. Let V be $\sigma_1 \sigma_2$ - closed set in (Y, σ_1, σ_2) . Since every $\sigma_1 \sigma_2$ - closed set is $(1, 2)^* - \delta_{gp}$ closed, V is $(1, 2)^* - \delta_{gp}$ closed in Y .

Since f is $(1,2)^*$ - δ gp irresolute, $f^{-1}(V)$ is $(1,2)^*$ - δ gp closed in X . Hence f is $(1,2)^*$ - δ gp continuous.

Remark 3.24. The converse of the above theorem is not true in general as shown in the following example.

Example 3.25. Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{a\}, X\}$, $\tau_2 = \{\emptyset, \{c\}, X\}$, $\sigma_1 = \{\emptyset, \{c\}, Y\}$ and $\sigma_2 = \{\emptyset, \{a\}, \{a, c\}, Y\}$. Let a map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then f is $(1,2)^*$ - δ gp continuous but it is not $(1,2)^*$ - δ gp irresolute because $f^{-1}(\{a, c\}) = \{a, c\}$ is not $(1,2)^*$ - δ gp closed in X .

Theorem 3.26. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ be two functions. Then

(i) $g \circ f : (X, \tau_1, \tau_2) \rightarrow (Z, \sigma_1, \sigma_2)$ is $(1,2)^*$ - δ gp continuous, if g is $(1,2)^*$ -continuous and f is $(1,2)^*$ - δ gp continuous.

(ii) $g \circ f : (X, \tau_1, \tau_2) \rightarrow (Z, \sigma_1, \sigma_2)$ is $(1,2)^*$ - δ gp irresolute, if g is $(1,2)^*$ - δ gp irresolute and f is $(1,2)^*$ - δ gp irresolute.

(i) $g \circ f : (X, \tau_1, \tau_2) \rightarrow (Z, \sigma_1, \sigma_2)$ is $(1,2)^*$ - δ gp continuous, if g is $(1,2)^*$ - δ gp continuous and f is $(1,2)^*$ - δ gp irresolute.

Proof. (i) Let A be any $\eta_1\eta_2$ closed set in (Z, η_1, η_2) . Since g is $(1,2)^*$ -continuous, $g^{-1}(A)$ is closed in (Y, σ_1, σ_2) . Also f is $(1,2)^*$ - δ gp continuous, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is $(1,2)^*$ - δ gp closed in (X, τ_1, τ_2) . Hence $g \circ f$ is $(1,2)^*$ - δ gp continuous function.

(ii) Let A be any $\eta_1\eta_2$ closed set in (Z, η_1, η_2) . Then A is $(1,2)^*$ - δ gp closed in (Z, η_1, η_2) . Since g is $(1,2)^*$ -irresolute, $g^{-1}(A)$ is $(1,2)^*$ - δ gp closed in (Y, σ_1, σ_2) . Also f is $(1,2)^*$ - δ gp irresolute, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is $(1,2)^*$ - δ gp closed in (X, τ_1, τ_2) . Hence $g \circ f$ is $(1,2)^*$ - δ gp irresolute function.

(iii) Let A be any $\eta_1\eta_2$ closed set in (Z, η_1, η_2) . Since g is $(1,2)^*$ -continuous, $g^{-1}(A)$ is $(1,2)^*$ - δ gp closed in (Y, σ_1, σ_2) . Also f is $(1,2)^*$ - δ gp irresolute, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is $(1,2)^*$ - δ gp closed in (X, τ_1, τ_2) . Hence $g \circ f$ is $(1,2)^*$ - δ gp irresolute function.

4. Conclusion

In this paper we define a new class of generalized continuous function called $(1,2)^*$ - δ gp continuous and investigate their relationships to other generalized continuous functions. We further study a new class of functions namely $(1,2)^*$ - δ gp irresolute. Also discussed some of their properties.

References

- [1] Benchalli.S and Toranagatti.B, “*Delta Generalized Pre-Closed Sets in Topological Spaces*”, International Journal of contemporary and Mathematical Sciences, Vol. 11, no. 6, 281-292.
- [2] Ekici.E and caldas.M, “*Slightly continuous functions*”, Bol. Soc. Parana. Mat.(3) 22(2004), no.2, 63-74.
- [3] Kelley.J.C, “*Bitopological Spaces*”, Proc. London math. Soc., 13(1963), 71-89.
- [4] Lellis Thivagar.M, Meera Devi.B, “*Bitopological B -Open sets*”, International Journal of Algorithms, Computing and Mathematics, Volume 3, Number 3, August 2010.
- [5] Lellis Thivagar.M and Nirmala Mariappan M, “*A note on $(1,2)^*$ - strongly generalized semi pre closed sets*”, Proceeding of International Conference on Mathematics and Computer sciences, 2010, 422-425.
- [6] Lellis Thivagar.M and Ravi.O, “*A Bitopological $(1,2)^*$ Semi generalized Continuous Maps*”, Bull. Malays. Math. Sci. Soc., (2) 29(1)(2006), 7988.
- [7] Lellis Thivagar.M and Ravi.O and Hatir.E, “*Decomposition of $(1,2)^*$ - continuity and $(1,2)^*$ - α continuity*”, Miskolc Mathematical notes, 2009, 10(2):163-171.
- [8] Long.E. P and Herington.L. L., “*Basic Properties of Regular Closed Maps*”, Rend. Cir. Mat. Palermo, 27(1978), 20-28.
- [9] Meera devi.B and Subbulakshmi.P, “ *$(1,2)^*$ - δ gp Closed Sets in Bitopological Spaces*”, Indian Journal of Research, Volume 06, Issue 07, July 2017, 27-32.
- [10] Raja Rajeshwari.R, Lellis Thivagar.M, Margaret nirmala.M and Ekici.E, “*A note on $(1,2)^*$ - gpr - closed set*”, Math . Maced,vol. 4 ,2006, 33-42.
- [11] Ravi.O and Lellis Thivagar.M and Jin, “*Remarks on extensions of $(1,2)^*$ - g - closed mapping in Bitopological spaces*”, Archimedes J.Math.1(2), pp.177-187, 2011.
- [12] Ravi.O and Lellis Thivagar.M, Kayathiri.K and Jopseph Isreal.M, “*Decompositions of $(1,2)^*$ - rg - continuous maps in bitopological spaces*”, Antarctica Journal Math.6(1)(2009), 13-23.
- [13] Sreeja.D and Janaki,C, “*ON $(1,2)^*$ - π gb - closed sets*”, International Journal of Computer Applications Volume 42- No.5, March ,pp0975 8887, 2012.
- [14] TantawayEl.A.O and Abu.DoniaM.H, “*Generalized Separation Axioms in Bitopological spaces*”, The Arab J.Sci.Eng., 30(1A), 117-129, 2005.